

Anoop,

My comments are below. Let me add two things –

1. My comments are random and by no means comprehensive. If I did not comment on some quote of mine, it does not mean that it is accurate. It only means that I didn't read it or didn't have the energy to correct it.
2. I'm putting this where I put (almost) everything that doesn't involve the privacy of others – in my Academic Pensieve. Yet this is a borderline case – a lot of it is my words, but a lot of it is also your words and work and privacy. If you want it removed let me know and it will be done immediately.

Best,

Dror.

The privacy thing is a joke - 2 seconds on google and I'll tell you who "Ben" is. Yet as I said, I couldn't come less.

The Finitist Mathematician: Ben

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"If it can be planned, it ain't research. Or at least, if it can be planned it's already half done, and hence it isn't the best of research. Thus my primary plan of mathematical research for the next few years is to *follow my nose*" ([emphasis in original]. "Ben", 2002).

Born in Kiryat Gat, a small town about 40 miles outside of Tel Aviv, Israel, in the 1960s, Ben is in his early 40s. He is a mathematician, at the rank of professor at a large urban university in Canada.

His elementary and secondary educations were in Israel, as well as his Master's of Science degree he earned in mathematics. His doctor in philosophy degree was earned at Princeton University in the early 1990s.

He described Israel, in relation to his upbringing, as a developing, and now developed country. He grew up in a secular, Westernized, non-religious household. His parent immigrated there, his father in the late 1940s, and his mother in the late 1950s. Ben has two older sisters, one by 10 years, and the other by ~~18~~ years, neither are mathematicians.

just 5.

His father was, in his words, "a low-level administrator." Both his parents were open-minded about science. His parents were happy he was reading math, "but," as he put it, "they could not help me because I was already ahead of them as a child." Ben recalls, however, that in order to keep him silent his father would have him digits of the phone book in the classified to add. Also, when Ben was in Grade 7 or 8, his parents bought him a "nice calculator, a TI-58...but the point is that it was unusual at the time." Ben said that he "really, really wanted it, and it was a significant expense." As he explained, his father not only had the expense but it was not easy to get; he had to wait for relatives to visit him from America. "So on some level," Ben said, "I got encouraged."

He grew up in a household that was much more connected to the state than Canada, and much richer than many around him. He described himself as lower middle-class. Adding some detail, however, he explained that in his town: “I was part of the upper ten percent of the population, well-to-do; however, if I was to go to Tel Aviv, I drop down to the lower middle class.” So, as he pointed out, his social economic status “is relative to the scale; at the time I was not aware that Israel was also as speck [like my small town was within the state].”

He ^{was} secular Israeli,” which he noted, “means quite a lot.” He ^{his family} was part of the Zionist movement, committed to the Jewish nationalist ideology, which he no longer relates. “Secular Israel still distinguishes itself from secular Canada,” he emphasized. After I suggested that the Jewish identity is strong because of the history of the Holocaust, he replied, “Your guess is absolutely right.” As he described the secular Jew: “It means no praying in the morning, no praying no matter how many times a day, but still the Jewish identity was very strong. But here I am describing my socio-cultural background, not me [specifically].”

In querying about the value of mathematics in his socio-cultural background, he said, “Zero, I mean you know the town I grew up in there was no one I can remember with interesting math. Zero is a good approximation.”

Ben was supported in his interest in mathematics in different ways. He first pointed out a lot depends on the year and the scale. “Basically, no one encouraged me to do mathematics, or science more generally,” as he put it, “but there was openness to it. I was extremely lucky because the schools I went to were open; they did not block it.” “I was a bit of an extreme student,” he went on, “some [grades] were 100 and some below 50, and I had a fair number of both. When you looked at

my grade point average it was about 75, but that is an average between many 100s and 50s.” The low grades related to things he had no interest in and subsequently spent little time on. The 100s were in mathematics where he also spent little time, because, he said, “I did not need to; so I passed with 100s without spending the time.”

As Ben recalled,

The bad thing I could say about school was that it was extremely boring. I felt like that I was serving a jail sentence. But to their [my teacher’s] benefit, they recognized it [that it was a jail sentence] and they let me graduate two years early. So basically they reduced my sentence. And that is exactly how I felt about it: that I am doing jail time and getting a reduction of my sentence. But I have to acknowledge that many more people suffer their jail sentence all the way to the end and thus suffer much more than me.

Thus, he finished high school at 16, not 18 as was common, even though his GPA was spotty.

“University was not at all jail, it was already fun,” he remarked however. Teachers, he pointed out, encouraged him in an unusual way: “They allowed me to part from them early, which is a significant point [laughter].”

Starting in a summer camp at the ~~Wiesmann~~ ^{Witzmann} Institute, between Grade 11 and 12, he first met mathematicians, “you know people who do real math, really think about math, not like high school...teachers that ~~that~~ ^{just} translate the content of the book.”

Up to the end of high school, the vast majority of kids, he said, “could not care less, mathematics, or science in general, was not conversation topic.” But, “the closer group to me, my own class, had no

interest mathematical conversation, yet they respected my interest in mathematics, science in general, or computers,” as he explained,” at the time it was computers.” However, the kids above him and different classes did stigmatize him. However, he added, “There was an even nearer groups, I did have a few friends that were scientifically-minded.” In sixth grade, he participated in science clubs, taking him to Be’er Sheva, the next nearest town to his own. “The very few that were close to me all have PhDs now, and by the way that is quite unusual, in my small town. My class had the most PhDs than those before or after.”

In university Ben said that he was not “exceptional in any way. I had lots of peers and to share mathematics with.” Incidentally, his wife is also a mathematician, who he met in the first year of university. It was computers and mathematics that interested him. And computers, at the time, required travelling to the next nearest town. And he wrote computer programs that were unusual for his age, beginning in Grade 7.

There was also an interested in electronics. “I learned little from my teachers. But they let me do my own thing...The class would do their own thing and I would sit on the side and play.” In school there was relatively little interaction with math, but they allowed me to participate outside of school.

“Already in sixth grade,” he pointed out, “I was sent to this computer’s enrichment club [at a local university], and then there was the summer camp, and the Israeli math Olympiad, which was not in school but at the time of school.”

In university, people were very open letting Ben take whatever classes he wanted. I queried, “What about Princeton?” He firmly remarked, “No, Princeton is a different story. I was done with classes...and the expectation is that you start doing research. And research is at different thing, at

different times, for different people, and in different moods.” He went on, “There is a component of extreme frustration,” he reflected. “Once reason it dawns upon you that what you are going to do at the best is going to be extremely insignificant; it’s going to be totally worthless at some level. Many of the people try, fail. Many people that succeed get a PhD, do not get a job. It’s [thesis work is] going extremely slow; your advisor does not appreciate you, the usual.”

Ben’s supervisor was a significant figure, but there was many others. I probed, “do you have your own style.” He replied, “Well, I definitely stole everything, but it was not just from him [my supervisor].” In any case, he noted, more generally, that no learning is “independent.”

The teaching style, up to the high school level, was “irrelevant,” said Ben. “The teaching style was happening outside of me; I was daydreaming inside of me.” “If I got A’s it was because it was easy. I can tell you what there was because I would occasionally peak out [from my daydreaming]. The teacher comes to board, homework, [and] the usual.” “University was the usual, big lectures early on, smaller classes later on, with little student participation in the lower classes a bit more in the higher classes,” he added. “I spent most of my time daydreaming, but I did study for examinations.”

In university he began to have relationships with professors. Up to university, ^{there} ~~there~~ were no teachers that were memorable to him, with the exception of one that was a master of chess; “without the chess, it would not be memorable...In university it was completely different.” There was perhaps 20 teachers that were memorable in university to Ben, being too long a list to start to go through. One example he gives is of an assistant professor without tenure—the first real mathematician he met that he learned “interesting math.”

The high school books were read “hardly...never dwelled upon.” But at “university I started reading

books...These type of questions [Anoop is asking (see Chapter 3, sec. "Interview Instrument")] sound like they are asked by the Ministry of Education...But the books I am talking about are not written by the Ministry of B.S. [bullshit], but live mathematicians; and there is nothing you can do about that. Some of it is good. Some of it is bad. Some of it is well-written and some not. I am not going to talk to you about what had wider margins or more pictures." So, in sum, up to high school it was, as he said, "a ~~tenured~~ jail sentence."

ten-year

Ben tells this anecdote, when asked if his ideas about learning were developed independently. His wife's advisor, a very distinguished mathematician, said that one of his favourite activities is to write book reviews. "He has complete freedom to say what he likes," according to Ben. "Because he [his wife's supervisor] was asked to do the review, hardly anyone reviews his work. He can start with two pages with whatever philosophical rambling he wants, then he makes half page to connect it to what is to be reviewed, then half a page of review and his job is done. But he got the freedom to speak as he wants." This story, though sort of off-track, does reinforce Ben's perchance for freedom.

Having two kids, Ben can now reflect upon mathematics teaching and learning from the perspective of a parent. One of his kids is in Grade 11 and is quite happy and enjoys being near the university his father teaches in, attending his lectures occasionally. The Grade 8 kid is more exceptional in mathematics than Ben was, by a margin; he is totally exempt from mathematics in school and his experience is much closer to his fathers, according to Ben.

Even at university, Ben said,

It was not that every course was great. There were lots of lousy courses and lousy

professors. But now they were individuals and I still expect them to be that way. If high school was extremely boring it was because the syllabus was set by the Ministry of Education...There is some room from the individuality of teachers, but it is so rigid. But at the university level, I still feel the class is much more the responsibility of the professor. I mean at the high school level the teacher is even told what words to use. At the university that is not the case.

Ben clarified that his experience is perhaps unique: professors tend to like top-level student. “So if you are a top-level student you are having a great time. On the whole if you are low-level student your experience is completely different.” As an aside, and providing an analogy, he pointed out that his experience of the city he teaches in is completely different of the people that live in the suburbs. So it is a matter of perspective: “If you are a C-level student you see a rigid system that does not care about you...well, I don’t know what they [C-level students] see, I’d have to ask.”

Once Ben was reading an advanced research article and knocked on a famous professors’ door: “Do you have a minute, I have a question?” This met with, from the professor’s side, “I am sorry, my office hours are...Perhaps a TA could...” Persisting, Ben pressed that he “only had a question about this particular sentence.” When the professor noticed it was one of his own research articles, he pronounced, “Oh, I’m sorry,” and went on to spend half an hour with him. “This still happens all the time,” Ben observed.

Beauty is something that appears in Ben’s description of his work repeatedly. For example he has written about [emphasis mine]:

Beautiful techniques.

The big dream of categorizing all of quantum algebra is too big for me. I remain convinced that that when fully drawn, *the picture will be beautiful*.

Behind every worthwhile argument there's *a beautiful picture* that fits together with everything else.

A beautiful picture always emerges.

And again, he spoke of the:

The beauty and depth of the theory.

So when asked about the role of beauty, Ben proclaims, “large,” in his choice of mathematics and the problems he sought within it. “I tend to move away things that come out ugly,” as he put it.

“Large computations with no fun story.” Like the epigram of this case indicates, Ben values “surprises” in his research.

Another feature of Ben's work is the persistent use of pictures. “I often think in pictures and like to tell my stories in pictures.” As he put it, “The working mathematician fears complicated words but loves pictures and diagrams.” In fact on his website he has an “Image Gallery” that includes pictures of knotted objects, symmetry, Jerusalem, Plants (like ferns), places, miscellaneous items, and symmetrical tiling. He conceded that the younger generation is more apt to use pictures because of their ease with technology, and that he probably uses more pictures than many mathematicians of his generation in his publications.

When faced with difficult problems in his field he uses a variety of strategies, too many to comment he observed: “If I knew how to solve to my problems, I would solve them. I write [to] myself [Appendix G: Samples of Questions from Ben's Notebooks], I draw pictures [Appendix E: Samples

of Pictures from Ben's Notebooks]. My notebooks are public, except when they relate to another person." In ~~face~~^{fact}, as we can see from his notebooks, he engaged in experimentation, in trial-and-error (see Appendix F: Samples of Experimentation from Ben's Notebooks).

When other students find mathematics difficult, Ben's response ran the gauntlet from "disrespect, to those people that do not try and have no effort, to admiration to those because of their ability or amazing ability to work." Even as a teacher, Ben feels this range of feelings depending on the students. "In a class of 100 students, there will be a few who will be crooks, who never wanted to open their minds, and more hopefully some that genuinely want to learn, a few quiet ones you can ever understand [smirks], and a few loud ones. There is no single answer."

Ben emphasized, "I am still as student, at a professor status." Thinking to himself, he said, "Perhaps I should put a list somewhere of things I don't understand somewhere. I don't understand quantum mechanics, but no one does." Ben found equilibrium between his interests and needs. "The practical need is to produce research. I cannot sit down all day and satisfy my own curiosity. Or maybe I can. But by definition, the fact that I have tenure proves I have not done that. You see, the system rejects people that are motivated by their own curiosity; it is called natural selection. Sorry, I have some cynical side. But it is true. But if you only want to solve the big problems of the universe you are likely not to get tenure, because you are likely not to solve them.

The two most interesting questions in mathematics, according to Ben, are the foundations of physics questions, and the foundations of computer science questions, "and I [Ben] have nothing to do with them." Ben described his interests as centering around three points. First, there is the research on algebraic knot theory, which I have already touched upon in my description of him (see Chapter 3,

sec. “Participants”). As Ben explained in his agenda for a first-year course on algebra he teaches thus: “To appreciate that the simplest is also the most fundamental.” In focusing on the basics, like algebra, he insisted that number theory is “just related to everything,” including knot theory. In fact, Leopold Kronecker (1823-1891), the German mathematician and logician, famously said, “God made the integers and all else is the work of man.” Ben wrote, modifying this, “God created knots all else in topology is the work of mortals.”

Adding a bit more, as he explained his idea for knot theory,

1. A map taking knots to be algebraic entities; such a map may be useful, say to tell different knots apart.
2. A collection of rules of the general nature of “if two knots are related in such and such a way, their corresponding algebraic entities are related in such and such a way. Such rules may allow us, say, to tell how far a knot is from the unknot or how far are two knots from each other.

Second, and related to the first point, he is interested in knot invariants, which I will not elaborate on it because it is a technical idea that takes us beyond our purposes here.

Finally, he believed that “computers are extremely valuable tool for mathematical research. So much of what we do is computable, and actually computing it very often leads to new conjectures and insights [in mathematics].” In fact, he said, “Thus in my opinion one of the biggest challenges in mathematics, perhaps the biggest challenge, is to turn computation from theory to practice...And gradually we all need to learn to appreciate computational rigour.”

As he pointed out, mathematics existed before computers. Parts of mathematics have nothing to do with computers and it “would be silly to program on a computer [some mathematics].” Ben explained,

Yet another part of mathematics by definition is about computation. So mathematics came because people wanted to compute certain things. You had a field three miles wide and you want to know [etc.]....So mathematics has been about computation from the start...and remain related to computation. However, because of various historical reasons, it is not computed. And a lot of the mathematical literature does not make the separation between what part is theory and what parts have practical value...There is no tradition of computing what is computable.

Some of this separation between mathematics and computation, he explained, has to do with the nature of learning. “My teachers did not have computers, so they only computed what they could do by hand...We tend to give the courses we took.” “There is no infrastructure” he also said. “Or I am lying, there is some, but it is lacking. Some of the reason is sociological. You get credit for new things. Making the mathematical infrastructure for the mathematics developed 100 years ago will unlikely get you credit.”

As Ben pointed out, the current programs for working on mathematics, like Mathematica™ and Maple are not adequate: “Being commercial, these programs are closed; we cannot inspect, verify, or modify their internals...It’s if Cauchy and Legendre had the copyrights on ϵ and δ , and the rest of us had to pay them fees to use ^{as} their notation and could use it only as prescribed by the original authors.”

Clarifying the nature of his research, however, he said, “the most interesting questions in math are not the most interesting questions in the world, [for example] how to bridge the gap between rich and poor...” According to Ben, the usefulness of mathematics gives rise to several cynical answers.

To begin with, he has pointed out that the most useful mathematics in everyday life is addition and subtraction, multiplication somewhat (e.g., “If I had 5 candles that cost \$1.29 and then I...”). He interjected, for instance, that carpenters (e.g., “You have to build a three by five gadget, how much wood do you need?”), and craftsmen in general, use mathematics a lot more than in him in daily life.

He observed that the typical person never uses powers in their life. He is unusual, he remarked, because he used powers to calculate his mortgage, when most people would consult a table.

However, powers “I have only used twice in my life,” he added.

He concluded, “Math is less and less useful the deeper you go. And that is a fact of life.” He went on, then, smiling,

The math you learn in elementary school is absolutely and truly useful. The math you learn in high school math is rarely very useful; the math you learn in university is even less useful; and the math you learn in graduate school is totally useless. And the math you do research on, God knows why people are doing it.

However, he added, “To a large extent the math that is taught is relevant...Calculus really is significant; I admit it is not as useful as addition and subtraction, but it is the next best thing.”

The cynical answer, to why mathematics is important, is that “it pays my bills, the more I do the

higher my salary becomes...There is also some curiosity. I really want to know the answers to the questions I ask. That may be delusional. To work on something you have to convince yourself that it is valuable or that it is interesting, regardless of the truth. Maybe that is what makes a good scientist, to be able to cheat oneself into thinking that one's work is important. But I have cheated myself; I do think my work interesting." As he observed, "I am extremely good at convincing myself that the things I write I care about." He added, "Sometimes the answers are technical... I want to know what happens if I add 16 to each side. Now I am making fun."

Between financial cynicism and pure curiosity, there is, according to Ben, cynicism at other levels. "I want to impress my peers." "I honestly do not know what is true, what is right or what's wrong [between the cynical and optimistic accounts of my motivation to do mathematics]."

Here is a most cynical answer Ben mustered at the social level:

Science is a conspiracy, the way university professors convince the government that there is value in what we do and they should invest in that; that the future of the country depends on that. And if you don't do that [invest in math and science] the other side will do that and have bigger bombs.

In the context of the cold war, it was the Russians, Ben observed, now it is the Japanese and the Koreans; "they'll have faster computers." "To tell you the truth," Ben added, "the gap between the math done to build bigger bombs or faster computers is quite large. So maybe you can point to one or two percent of the research does contribute to such causes, but the vast majority doesn't."

There is another side to this conspiracy, according to Ben: "It is a pyramid scheme. The professors

are paid well and having good lives; and the students want to be there. It's a classical pyramid scheme. They pay our salaries because we promise them a dream."

There is truth to some of cynical views, according to Ben, but "if we do not push forward [doing math research] we degenerate...Basically, we have small number of books [consequently] that becomes [like] a Bible, and we teach less and less well, and with less and less understanding."

Further, according to Ben, "If every math ^{teacher} has not had contact with research, and that needs a huge infrastructure, we will start losing what we have. It is not for me to put the weight between these views and the cynical views of before." There is also the case of "what may be useful 50 or 100 years from now." Ben concluded, "This is a relatively cheap expense, all this math research is less expense than sending a man to the moon, and it is not clear sending the man to the moon achieved anything more."

Ben's ideas about learning are not based on studying formal theories of learning. "I never had taken a teaching class, no formal training [in teaching]." "The only way I know something is if I add to it," Ben observed. The only way I understand something in mathematics, is if I do something further, something more than the textbook. The only way I ever learn something is by being creative. And this creativity can take many forms. Sometimes it can be writing a computer program, finding analogies, using it in an another context, sometimes it is sitting for an hour and thinking about it, till a mental image of the subject appears, and that was not in the textbook. And it is very hard if you have not experienced it...I will only understand it once I have my own image it.

However, according to him, "This contradicts our system of rewarding for achievement...Every week new material is covered and for a reason...If we left time for students to reflect they would use

it [reflection time] to go partying...It is not something the university can give you, it is in you...there is no way to encourage it [reflection], unless it is perhaps a graduate student you meet regularly.”

“Most students do not know what it means to be creative,” said Ben. “And I should say only I discovered it relatively late. The idea of doing more ^{than} reviewing what was in the blackboard came to me in second year of university.” He goes on, “If I tell my students to reflect, it would be a 95% failure; as a student in the faculty of education, you [Anoop] would have to categorize it as a failure.”

Ben said his ideas about learning have evolved, but there has never been a revolution. As he pointed out, “You know, there is a completely different teaching philosophy that goes, you memorize everything; and there is a very strong case in favour of it. Pick a book.” I query, “Do you want me to get it.” “No.” Looking around his office, I blurt out, “*Abstract Algebra*.” Ben quips, “No, that’s too easy.” So, I suggest “*Quantum Field and Strings*.” “Okay,” he continued. He explained, then, that if he had memorized *Quantum Field and Strings*, he could think about it while riding the bus or seeing a clouding formation in the sky and it will remind him “of page 422.” “The idea is that you will grow into understanding,” he explained. “I know people who this [memorization] works. For me, I am weak at that. I am not good at memorizing, I add my creativity as a learn. For some people this [memorization] is the right way to learn.”

So, says Ben, “I try to entertain my students, having giving up on trying to educate them. Sorry, now you are seeing my cynical side. When you teach a class, there is an aspect of circus, and comedy. There is an aspect of being a stand up comedy type. I am not saying I am good at that; most of the time I fail.”

More seriously, in his “Teaching Philosophy,” he takes the fictional persona of a weak student, and aims to teach to him:

*you have it wrong.
This really is me.*

I am a horrible student and horrible listeners...It's very difficult for me to pay attention...if the purpose is not absolutely clear to me...I assume their [?] is no purpose and fall asleep. If the route to the goal seems too tedious, I'm quick to lose my patience. I sense right away if the speaker doesn't fully understand what they are talking about...If the language is not clear, I don't listen. If her writing is not readable, I don't bother reading...It all gets worse if I don't have sympathy for the speaker as a person; if I don't get the signal that (s)he really wants me to listen (ME; not just somebody else in the room). And I'm shallow; jokes, anecdotes and (well performed) technological slights of hand grab my attention and make me sympathetic... finally, I forget everything I have heard very quickly unless there is a clear and meaningful moral to it and the logical structure leading to it is simple, motivated, clear and free of unnecessary complications. When I lecture or teach I try to satisfy...[Ben], assuming he is in the audience...I always try to tell my students about the big picture and where to find it (but only few of them take notice).

From these excerpts, we can glean what Ben values in teaching: transparency of purpose; expertise; clear language, spoken and written; humanity; and meaningfulness to larger contexts—the big picture. I inquired, “What would you like your students to achieve as a mathematics educator?” He said, “There is no single goal for every student.” He conjectured, “Happiness!” He went on, “For some of them it would be career advancement, for some of them it would be to satisfy their curiosity, for some of them to do research like I am doing, but I really can't say.”

I turned to ask about the relationship between this work and the socio-cultural context of his upbringing, Ben referred to the famous last lecture by Randy Pausch, a professor of computer science at Carnegie Mellon, diagnosed with cancer, who spoke about achieving his childhood dreams. Pausch spoke of how he wanted to visit Disney Land, and how he eventually did and was also able to contribute to that place professionally. Ben's sister pointed out to him, however, that there is something disturbing about his lecture because it makes you think that you should try to achieve your childhood dreams. "What about your dreams when you are 30," Ben said. "In some sense it is childish to want to achieve your childhood dreams. When I was three, I wanted to be astronaut; now does it makes it sense I still want to risk my life for no purpose whatsoever." So he concluded that ~~his upbringing is irrelevant to his current research. He hopes to achieve his research goals by "working hard, get lucky, and it would be helpful to have a few fewer administrative duties."~~ ~~XX~~

Ben described himself as a "philosophical extreme in math...I do not believe what I can't calculate [see Appendix H: An Example of Philosophical Remarks from Ben's Notebooks]. There is a whole spectrum of disbelief, as he pointed out." Ben remarked,

- a) It is valuable if you can compute them [the numbers] quickly
- b) Less valuable if you compute them less quickly
- c) Much less valuable if you can theoretically compute them (but no computer could actually do so)
- d) No value if you can never compute them

e) No [significant mathematical] value if they are formal consequences of a system

As he pointed out, relating to (e), Gödel's incompleteness theorem (i.e., the idea that any finite formal system like Peano Number Theory gives rise to statements that are not provable within in it) is not very interesting mathematics, but makes up for it because of its philosophical value. He aligns himself, accordingly, with finitism and intuitionism, the idea, in slogan form and as he put it, that "if you can program it, it is finite."